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SPIN-ORBIT EFFECTS ON SINGLE PARTICLE TRANSITIONS IN QUANTUM WELLS AND RINGS

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Resumen

Se considera un anillo cuántico con ancho finito, sujeto a la acción de un campo magnético externo aplicado en la dirección transversal cuando ambos tipos de interacción espín-órbita Rashba-Bychkov y Dresselhauss son tenidas en cuenta. Para estudiar las transiciones de una sola partícula, se extendió el método propuesto por Lipparini et al. para pozos cuánticos al caso de confinamiento unidimensional fuerte. Expresiones analíticas para los estados de una partícula cuasi up y cuasi down y sus energías han sido obtenidas. El nivel de transición de una sola partícula inducida por un campo electromagnétco ac ha sido considerado y los elementos de matrix concernientes a estas transiciones relacionadas con la resonancia ciclotrónica (qup-qup y qdown-qdown) y las transiciones epin-flip han sido calculadas.

Palabras Claves: *Interacción Rashba- Bychkov, Interacción Dresselhauss, Anillo cuántico*

Abstract

It is considered a quantum ring with finite width, subjected to the action of an external magnetic field applied in the transversal direction when both Rashba-Bychkov and Dresselhauss interactions are taken into account. In order to study the single particle transitions, we extend the method proposed by Lipparini et al. for quantum wells, to the case of strong 1D confinement. Analitical expressions for quasi up and quasi down single particle states and energies have been obtained. Single particle level transitions induced by ac electromagnetic fields have been considered and the matrix elements concerning the allowed transitions related to cyclotron resonance (qup-qup and qdown-qdown) and spin-flip transitions have been calculated.

Key words: *Rashba- Bychkov interaction, Dresselhauss interaction, Quantum Rings*

1. INTRODUCTION

Considerable attention has been devoted to research concerning the control and manipulation of the spin-orbit (SO) interaction in semiconductor nanostructures [1-7]. In this sense, it is important to understand the way how the application of external fields can affect the electronic properties of particles under the conditions of strong confinement. In this report we consider a quantum ring with circular cross section, subjected to the action of an external magnetic field applied in the transversal direction when both Rashba-Bychkov and Dresselhauss interactions are taken into account. In order to study the single particle transitions, we extend the method proposed by Lipparini et. al for quantum wells [5], to the case of a finit width ring with strong 2D confinement.

This work is organized as follows. In Sec. 2 we present the general formalism for the single-particle (sp) Hamiltonian. These results are used in Sec. 3 to study the transitions caused by an external electromagnetic field.

2. THEORETICAL FRAMEWORK

We consider an electron of charge e and effective mass m in a (2D) ring in the plane $z = 0$, centered on the z axis, with external radius ρ_{Ex} and inner radius ρ_{In} under an uniform magnetic field B normal to the plane of the ring sample. The single particle Hamiltonian H_0

$$H_0 = \frac{\mathbf{P}^2}{2m^*} + \frac{1}{2} g\mu_B B\sigma_z + \frac{\lambda_R}{\hbar}\left(\sigma_x P_y - \sigma_y P_x\right) + \frac{\lambda_D}{\hbar}\left(\sigma_x P_x - \sigma_y P_y\right) \quad (1)$$

contains the following terms: *(i)* kinetic; *(ii)* Zeeman; *(iii)* Rashba; *(iv)* Dresselhauss, where $\boldsymbol{P}=-i\hbar\nabla+(e/c)\boldsymbol{A}$, $\boldsymbol{A}=(1/2)B\rho\boldsymbol{e}_\varphi$, g is the effective gyromagnetic factor, μ_B is the Bohr magneton and σ_i ($i=x, y, z$) are the Pauli matrices. Schrödinger equation $H_0|\phi\rangle=\hbar\omega_c\varepsilon|\phi\rangle$ can be solved by expanding the components $|\phi_1\rangle$, $|\phi_2\rangle$ of the two-component spinor $|\phi\rangle$ in terms of the electron states in absence of spin-orbit effects as $|\phi_{a,m}(\xi,\varphi)\rangle=\Sigma c_{a,m}\exp(im\varphi)R_{a,m}(\xi)$, where a=1,2, m=0,±1, ±2…, $R_{a,m}(\xi)=[M(\alpha_a, |m|+1,\xi)-\Lambda_a U(\alpha_a, |m|+1,\xi)]\xi^{|m|/2}\times\exp(-\xi/2)$, $\Lambda_a = M(\alpha_a,b,\xi_{In})/U(\alpha_a,b,\xi_{In})$, $\xi_{In} = Q\rho_{In}^2/\bar{\rho}^2$, $\bar{\rho}=(\rho_{Ex}+\rho_{In})/2$, $Q=\Phi/\Phi_0$, α_a are the roots of $R_{a,m}(\xi_{Ex})=0$, M and U are the solutions of the Kummer equation. We obtain

$$\left(\frac{\varepsilon_1^+}{\hbar\omega_c} - \varepsilon\right)c_{1,m} + \tilde{\lambda}_R \tilde{A}_{2,m+1}c_{2,m+1} - i\tilde{\lambda}_D \tilde{A}_{2,m-1}c_{2,m-1} = 0 \quad (2)$$

$$\left(\frac{\varepsilon_2^-}{\hbar\omega_c} - \varepsilon\right)c_{2,m} - \tilde{\lambda}_R \tilde{A}_{1,m-1}c_{1,m-1} - i\tilde{\lambda}_D \tilde{A}_{1,m+1}c_{1,m+1} = 0 \quad (3)$$

where $\tilde{\lambda}_{R,D} = \left(2\sqrt{Q}/\hbar\omega_c\bar{\rho}\right)\lambda_{R,D}$,

$$\frac{\varepsilon_{0,m}^{\pm}}{\hbar\omega_c} = -\alpha_a \pm \frac{1}{4}g + \frac{1}{2}\left(|m| + m + 1\right),$$

$$A_{am}^{\pm} = \left(\int_{\xi_{In}}^{\xi_{Ex}} \tilde{R}_{a,m}^{\pm}(\xi)R_{a,(m\pm1)}d\xi\right)N_m^{-1}, N_m = \int_{\xi_{In}}^{\xi_{Ex}} \xi R_{a,m}^2(\xi)d\xi,$$

$$\tilde{R}_{a,m}(\xi) = 2\sqrt{\xi}\frac{\partial R_{a',m}(\xi)}{\partial\xi} \pm \left(\frac{m}{\sqrt{\xi}} + \sqrt{\xi}\right)\times R_{a',m}(\xi).$$

Following Lipparini *et al.* [5], analytical expressions for quasi up and quasi down single particle states and energies are obtained in the limit $\tilde{\lambda}_{R,D} << 1$ by considering that for each state $|\phi_{a,m}(\xi,\varphi)\rangle$ spin orbit interaction is allowed to couple it only with states $|\phi_{a,m\pm1}(\xi,\varphi)\rangle$. For the case of quasi up (qup) states, we obtain

$$\begin{pmatrix} |\phi_1^u\rangle \\ |\phi_2^u\rangle \end{pmatrix} = \sum_m \begin{pmatrix} c_{1,m}|\phi_{1,m}\rangle \\ c_{2,m-1}|\phi_{2,m-1}\rangle + c_{2,m+1}|\phi_{2,m+1}\rangle \end{pmatrix}, \quad (4)$$

$$\varepsilon_{up}^- = -\alpha + \frac{1}{4}g + \frac{1}{2}\left(|m| + m + 1\right) - \frac{2}{g}\tilde{A}_{1,m}^{\pm}\left(\tilde{\lambda}_R^2\tilde{A}_{2,m+1} + \tilde{\lambda}_D^2\tilde{A}_{2,m-1}\right), \quad (5)$$

where

$|c_{1,m}| \approx 1 - 2\frac{\tilde{A}_{1,m}^{2\pm}}{g^2}\left(\tilde{\lambda}_D^2 + \tilde{\lambda}_R^2\right)$, $c_{2,m+1} = -2\tilde{\lambda}_R\tilde{A}_{1,m}^{\pm}c_{1,m}/g$, $c_{2,m-1} = -2i\tilde{\lambda}_D\tilde{A}_{1,m}^{\pm}c_{1,m}/g$.

Similar expressions for quasi down (qdown) states are also obtained.

$$\begin{pmatrix} |\phi_1^d\rangle \\ |\phi_2^d\rangle \end{pmatrix} = \begin{pmatrix} c_{1,m-1}|\phi_{1,m-1}\rangle + c_{1,m+1}|\phi_{1,m+1}\rangle \\ c_{2,m}|\phi_{2,m}\rangle \end{pmatrix}, \quad (6)$$

$$\varepsilon_{down}^+ = -\alpha - \frac{1}{4}g + \frac{1}{2}\left(|m| + m + 1\right) + \frac{2}{g}\tilde{A}_{2,m}^{\pm}\left(\tilde{\lambda}_R^2\tilde{A}_{1,m-1} + \tilde{\lambda}_D^2\tilde{A}_{1,m+1}\right), \quad (7)$$

where

$|c_{2,m}| \approx 1 - \frac{2}{g^2}\tilde{A}_{2,m}^{2\pm}\left(\tilde{\lambda}_R^2 + \tilde{\lambda}_D^2\right)$, $c_{1,m+1} = \frac{2}{g}i\tilde{\lambda}_D\tilde{A}_{2,m}^{\pm}c_{2,m}$, $c_{1,m-1} = -\frac{2}{g}\tilde{\lambda}_R\tilde{A}_{2,m}^{\pm}c_{2,m}$.

We can use the preceding results to study the transitions induced in the system by the interaction with a left circular-polarized electromagnetic wave propagating along the z direction, i.e., perpendicular to the plane of motion of the electrons, whose vector potential is $\boldsymbol{A}(t)=2A(\cos\theta,\text{sen}\theta)$ with $\theta=\omega t-qz$, in the dipole approximation ($q\approx0$).

The single particle (sp) interaction Hamiltonian $\mathbf{j}.\mathbf{A}/c+g\mu_B\mathbf{s}.(\nabla\times\mathbf{A})$, where $\mathbf{j}=e\mathbf{v}/\sqrt{\varepsilon}$ reads

$$h_{\text{int}} = \frac{1}{c}\frac{eA}{\sqrt{\varepsilon}}\left[v_-\exp(i\theta) + v_+\exp(-i\theta)\right] + \frac{g^*\mu_B Aq\hbar}{2}\left[\sigma_-\exp(i\theta) + \sigma_+\exp(-i\theta)\right], \quad (8)$$

where $v_\pm = \frac{P_\pm}{m} \pm i\frac{\lambda_R}{\hbar}\sigma_\pm + \frac{\lambda_D}{\hbar}\sigma_\mp$.

The Hamiltonian h_{int} can be rewritten as

$$h_{\text{int}} = \frac{eA}{c\sqrt{\varepsilon}}\left[\alpha_-\exp(i\theta) + \alpha_+\exp(-i\theta)\right] + \frac{1}{2}Ag\mu_B q\hbar\left[\sigma_-\exp(i\theta) + \sigma_+\exp(-i\theta)\right], \quad (9)$$

where the operators α_+ y α_- acting on the spinor $|\phi_{a,m}\rangle$ are

$$\alpha_+ = \begin{pmatrix} \frac{1}{m}P_+ & i\frac{2\lambda_R}{\hbar} \\ \frac{2\lambda_D}{\hbar} & \frac{1}{m}P_+ \end{pmatrix}, \quad \alpha_- = \begin{pmatrix} \frac{1}{m}P_- & \frac{2\lambda_D}{\hbar} \\ -i\frac{2\lambda_R}{\hbar} & \frac{1}{m}P_- \end{pmatrix}. \qquad (10)$$

3. RESULTS AND DISCUSSION

Due to the fact that $g<0$, the lowest energy level in GaAs rings is the quasi up one at an energy, containing both Rashba and Dresselhauss contributions, which is in contrast with the behavior of such level in quantum wells, containing only the Rashba contribution [5].

In Fig.1, we want to show that for a given value of the strength of Dresselhauss interaction ($\tilde{\lambda}_D^2 = 0.01$), the splitting $\delta E_m^u = \left[\left(\varepsilon_{up}^+ - (\varepsilon_{0,m}/\hbar\omega_c\right)\right]$ of the single particle energy due to spin-orbit effects increases with the square of the intensity of the Rashba term. It is seen that spin-orbit effects do not alter the period of Aharonov-Bohm oscillations in 1D quantum rings, but it is observed a shift toward the left of the degenerated points. It is important to remark that there exists a set of values of the applied magnetic field for which the shift of the SO oscillations disapears. This fact can motivate the search of new electron transport mechanisms as it is discussed by Fabrízio et. al [6].

In Fig.2, we illustrate the role of the Zeeman effect on the energy of quasi up (qup) and quasi down (qdown) states as a function of the magnetic field for a quantum ring of finite width of dimensions ρ_{In} = 5a$_B$ and ρ_{Ex} = 20a$_B$. It is seen that the Zeeman effect increases the energy of the qdown state for m=0,1,-1 (dashed lines), and decreases the energy of the qup state for m=0,1,-1(thick lines) with respect to the corresponding states in the absense of Zeeman effect (thin lines).

In Fig.3, we show the splitting of the energies for the qup and qdown states due to the SO interaction as a function of the magnetic field for a quantum ring of finite width of dimensions ρ_{In} = 5a$_B$ and ρ_{Ex} = 20a$_B$. It is seen that for the qup states (Fig3a) the energy splitting increases linearly with the magnetic field for m=01,-1, and for the qdown states (Fig.3b) it decreases linearly with the magnetic field for m=0,1,-1.The inset to Fig.3a shows a zoom of the qup and qdown energies for a wider range of magnetic fields. It is noted that the corresponding monotonous behaviour of the splitting of the ground state energy with the magnetic field is in contrast with the corresponding oscillatory behaviour of such splitting in 1D quantum rings.

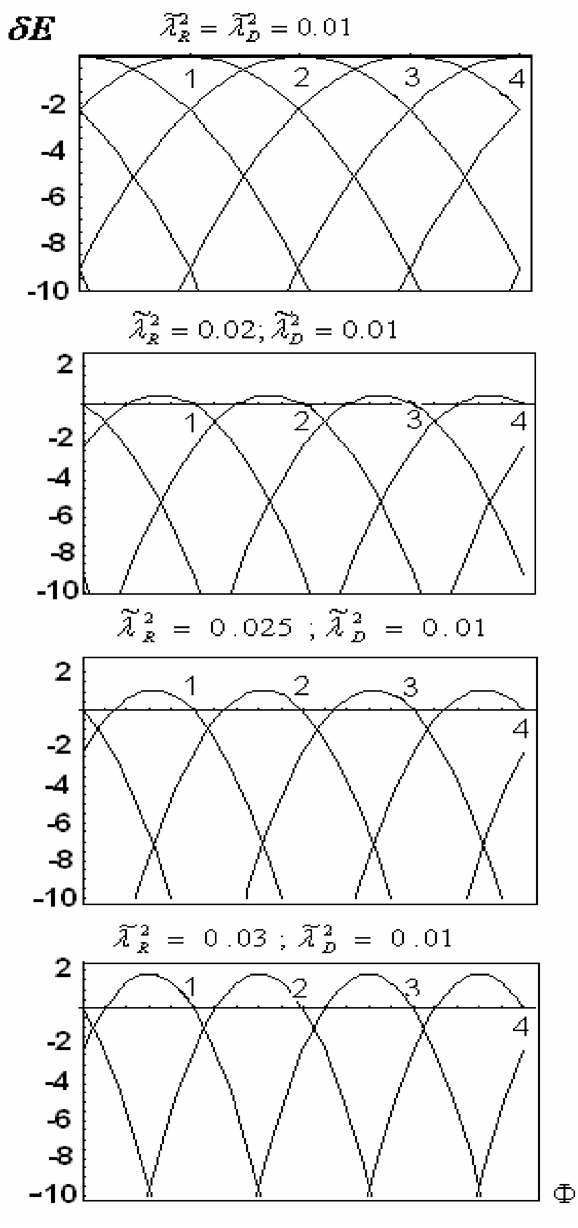

Figure 1. Splitting of the lower single particle quasi-up energies in GaAs 1D quantum rings δE (in units of $(\hbar^2/2m\rho^2)*10^{-4}$) as a function of the magnetic flux Φ (in units of the quantum of magnetic flux Φ_0) for a fixed value of the strength of Dresselhauss interaction ($\tilde{\lambda}_R$=0.01) and for: *(a)* $\tilde{\lambda}_R$=0.0, *(b)* $\tilde{\lambda}_R$=4λ_D. Calculations are performed for g = -0.44.

We consider next several useful examples of sp matrix elements involving the operators α_+ for a 1D quntum ring, which are proportional to v_+ and σ_-, and the qup and qdown sp states of Eqs. 4 and 6.

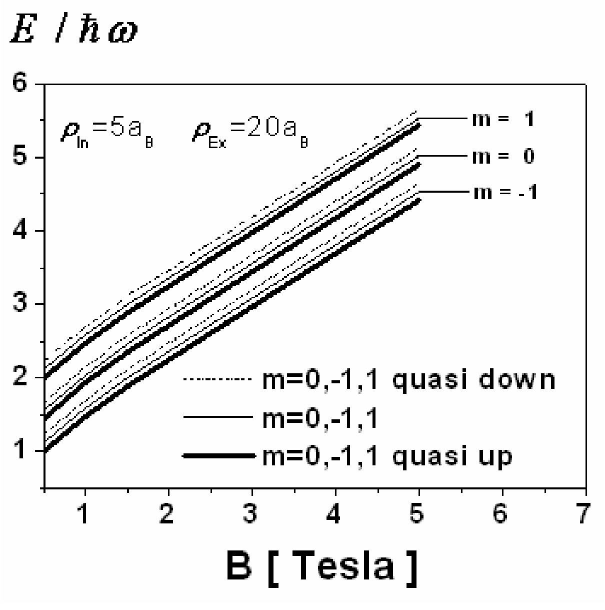

Figure 2. Splitting of single particle energies of a charge carrier in a quantum ring of finite width of dimensions ρ_{In} = 5a$_B$ and ρ_{Ex} = 20a$_B$ as a function of the applied magnetic field. The thin continue lines correspond to states in the absense of Zeeman effect.

It is important to distinguish between qup-qup, qdown-qdown, qup-qdown, and qdown-qup transitions. The qup-qup and qdown-qdown transitions represent the usual the cyclotron resonance (CR), and the qup-qdown and qdown-qup are related to spin-flip transitions.

The qup-qup and qdown-qdown transitions, up to the order $\widetilde{\lambda}^2_{R,D}$, are dominated by the transition $n \to n+1$ at the energies $E^u_{n+1} - E^u_n$ and $E^d_{n+1} - E^d_n$ with matrix elements

$$\left|\langle (n+1)_d \left|\alpha_+\right| n_d \rangle\right| = \left|\langle (n+1)_u \left|\alpha_+\right| n_u \rangle\right| \approx \frac{\hbar}{mR}\left(n+1+\frac{\Phi}{\Phi_0}\right); \qquad (11)$$

the corresponding energy splitting is

$$\delta E = \left(E^u_{n+1} - E^u_n\right)Q = \frac{1}{4}\left(n+\frac{1}{2}+Q\right)+\frac{1}{g}\left[\widetilde{\lambda}^2_R(n+1+Q)+\widetilde{\lambda}^2_D(n+Q)\right] \qquad (12)$$

The qup-qdown and qdown-qup transitions with energy $E^d_n - E^u_n$ have the matrix element

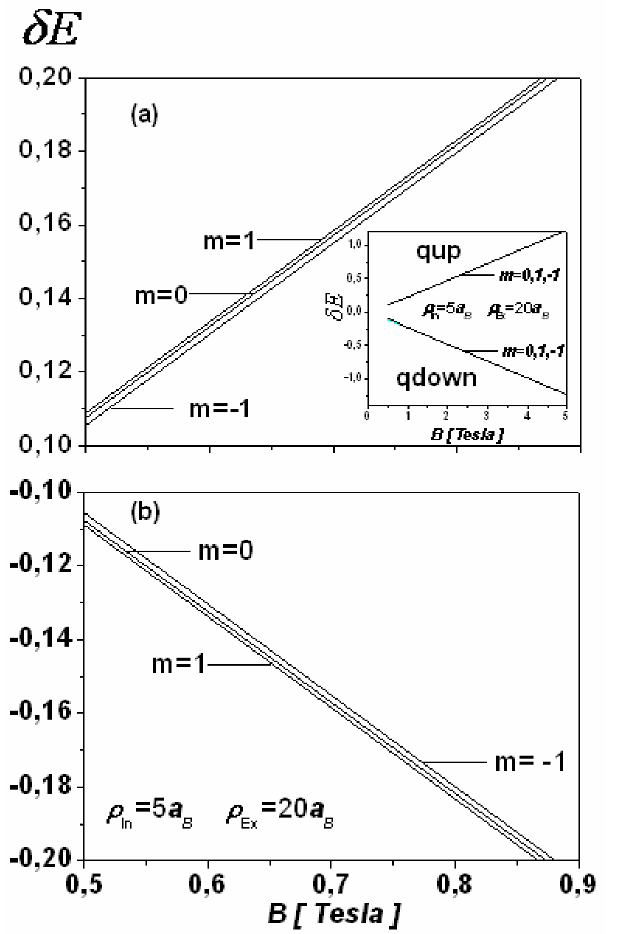

Figure 3. Spin-orbit splitting on the energy (in units of $(\hbar^2/2m\rho_{Ex}^2)*10^{-4}$) of some quasi up (figure 3a) and quasi down states (figure 3b) for a confined carrier in a quantum ring of finite wide as a function of the magnetic field.

$$\left|\langle n_d \left|\alpha_+\right| n_u \rangle\right| = \left[\frac{2R}{\hbar}\sqrt{\frac{1}{4Q}}\hbar\omega_c + \frac{2\hbar}{mRg}\sqrt{\frac{1}{4Q}}(n+Q)\left[n+Q+\frac{1}{2}\right]\right]\widetilde{\lambda}_D \qquad (13)$$

The corresponding energy splitting is

$$\delta\varepsilon = \left(\varepsilon^d_n - \varepsilon^u_n\right)Q = -\frac{gQ}{2} - \frac{1}{g}(n+Q)^2\left[\widetilde{\lambda}^2_R + \widetilde{\lambda}^2_D\right]. \qquad (14)$$

Note that the transition matrix element is linear in $\widetilde{\lambda}_D$, and that in the presence of the Rashba interaction alone, α_+ causes no spin-flip transition, which is in agreement with the result obtained by Lipparini *et al.* for quantum wells[5]. Additionally, spin-flip excitations are also possible at an energy $E_{n+1}^{d} - E_{n}^{u}$ with matrix element $|\langle n_d | \sigma_- | n_u \rangle| \approx 2$, which is also in agreement with similar result for quantum wells.

4. CONCLUSION

We have studied the influence of SO interactions on the electronic spectrum of confined carriers in quantum rings. It was shown that the magnetic field behavior of the ground state energy differs considerably in 1D rings when compared with rings with finite width. It was found that in 1D quantum rings the lowest energy state depends on both Rashba and Dresselhauss terms, which differs considerably with the behaviour of such state in quantum wells, where only the Rashba contribution is present. It was found also that the shift of the SO oscillations in 1D rings can disappear.for a special set of values of the applied magnetic field. Finally, it was demonstrated that Rashba and Dresselhauss contributions can control the transitions between energy states involving spin flip transitions.

5. ACKNOWLEDGMENTS

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